Theoretical Relationship Between Hot-Side Temperature (T_h) and Maximum Temperature Difference (ΔT_{max})

A First-Principles Derivation for Thermoelectric Cooler Modules

Analog Technologies, Inc. | Technical Note TNTECM-012 | April 2020

1. Introduction

In thermoelectric cooling (TEC) modules, the maximum achievable temperature difference ΔTmax is not a fixed constant — it depends on the absolute hot-side temperature Th at which the module operates. Engineers frequently observe that a TEC rated for ΔTmax ≈ 68 °C at Th = 27 °C will reach roughly 76 °C at Th = 50 °C and over 96 °C at Th = 100 °C. The hardware has not changed; the physics simply favors warmer operation.

This Technical Note derives the relationship from first principles, then explains in plain language why higher Th produces a larger ΔTmax. A short cartoon and a single intuitive analogy capture the mechanism. Figure 1 plots the predicted curve, normalized to the datasheet reference (Th = 27 °C), together with the optimum drive current ITEC also normalized to its datasheet value Imax.

2. The Thermoelectric Energy Balance Equation

The net cooling capacity Qc at the cold junction is governed by three competing physical phenomena:

Peltier cooling — active heat pumping proportional to current I and cold-side absolute temperature Tc: Qpeltier = α · Tc · I. This term is the root cause of the Th-dependence.

Joule heating — parasitic resistive dissipation I^2R inside the pellets; one half flows back into the cold side.

Fourier conduction — parasitic thermal back-conduction K · ΔT from hot to cold side through the pellets.

The cold-side energy balance is therefore:

$$Qc = \alpha \, Tc \, I - \tfrac{1}{2} I^2 R - K \, \Delta T \quad \textbf{(1)}$$

where *α* is the Seebeck coefficient (V/K), *Tc* is the cold-side absolute temperature (K), *I* is the operating current (A), *R* is the electrical resistance (Ω), *K* is the thermal conductance (W/K), and ΔT = Th − Tc.

3. Deriving ΔTmax — Optimum Drive Current

ΔTmax occurs when Qc = 0 — the operating point where all of the module's pumping capacity is consumed by its own internal losses. Setting Qc = 0 in Equation (1):

$$0 = \alpha \, Tc \, I - \tfrac{1}{2} I^2 R - K \, \Delta Tmax \quad \textbf{(2)}$$

Solving for ΔTmax as a function of I:

$$\Delta Tmax = (\alpha \, Tc \, I - \tfrac{1}{2} I^2 R) \; / \; K \quad \textbf{(3)}$$

Optimize the current by differentiating Equation (3) and setting d(ΔTmax)/dI = 0:

$$d(\Delta Tmax)/dI = (\alpha\, Tc - I\, R) / K = 0 \quad \textbf{(4)}$$

which immediately gives the optimum current:

$$Iopt = \alpha\, Tc / R \quad \textbf{(5)}$$

Note: Iopt scales linearly with Tc, so the optimum drive current itself rises with Th. Figure 1 plots ITEC / Imax (right axis), normalized to the datasheet reference at Th = 27 °C.

Substituting Iopt back into Equation (3) and simplifying:

$$\Delta Tmax = ½ \cdot (\alpha^2 / R\, K) \cdot Tc^2 \quad \textbf{(6)}$$

4. The Figure of Merit Z and the Cooling-Side Master Equation

The grouped material parameter α^2 / (R K) is the thermoelectric Figure of Merit Z, with units of K^{-1}:

$$Z = \alpha^2 / (R\, K) \qquad [K^{-1}] \quad \textbf{(7)}$$

Equation (6) collapses to the master form for the cooling side:

$$\Delta Tmax = ½ \cdot Z \cdot Tc^2 \quad \textbf{(8)}$$

This is the central result of the derivation. It states that the maximum temperature difference scales linearly with Z and with the *square of the cold-side absolute temperature*. For Bi_2Te_3 at room temperature, Z ≈ 2.5 × 10^{-3} K^{-1}.

5. Why Higher Th Helps — the Physics in One Sentence

Equation (8) is in terms of *Tc*, but engineers control Th. Because ΔT = Th − Tc, raising Th raises Tc as well — even at the maximum-ΔT operating point. And the master equation says cooling power scales with the *square of Tc*. So:

higher Th → higher Tc → more Peltier heat per amp → larger ΔTmax

6. An Intuitive Analogy — Ants Carrying Heat Buckets

Picture each electron as a tiny ant carrying a bucket of heat from the cold side up a flight of stairs to the hot side. The bucket capacity is set by physics: each ant can carry an amount of heat proportional to the absolute temperature where it picks up the load — $qe = \alpha \cdot Tc$ per coulomb (entropy times temperature, the Peltier cooling rate per amp).

On a cold day (low Th, and so a low Tc), each ant's bucket is small. The ant struggles up the stairs and barely moves any heat per trip; the module's pumping margin is modest, and ΔTmax stays around 68 °C.

On a hot day (high Th, and so a high Tc), each ant carries a much larger bucket. Same number of ants (same drive current I), but more heat per trip overcomes more parasitic loss before the cooling capacity is exhausted. ΔTmax climbs to roughly 92 °C — even though the hardware is identical.

That is the entire mechanism behind ΔTmax = ½ Z Tc². The cooling-side master equation says cooling power scales with the square of Tc, and Tc rises with Th, so warmer ambient operation lets each electron move more heat per trip. Figure 2 shows the picture.

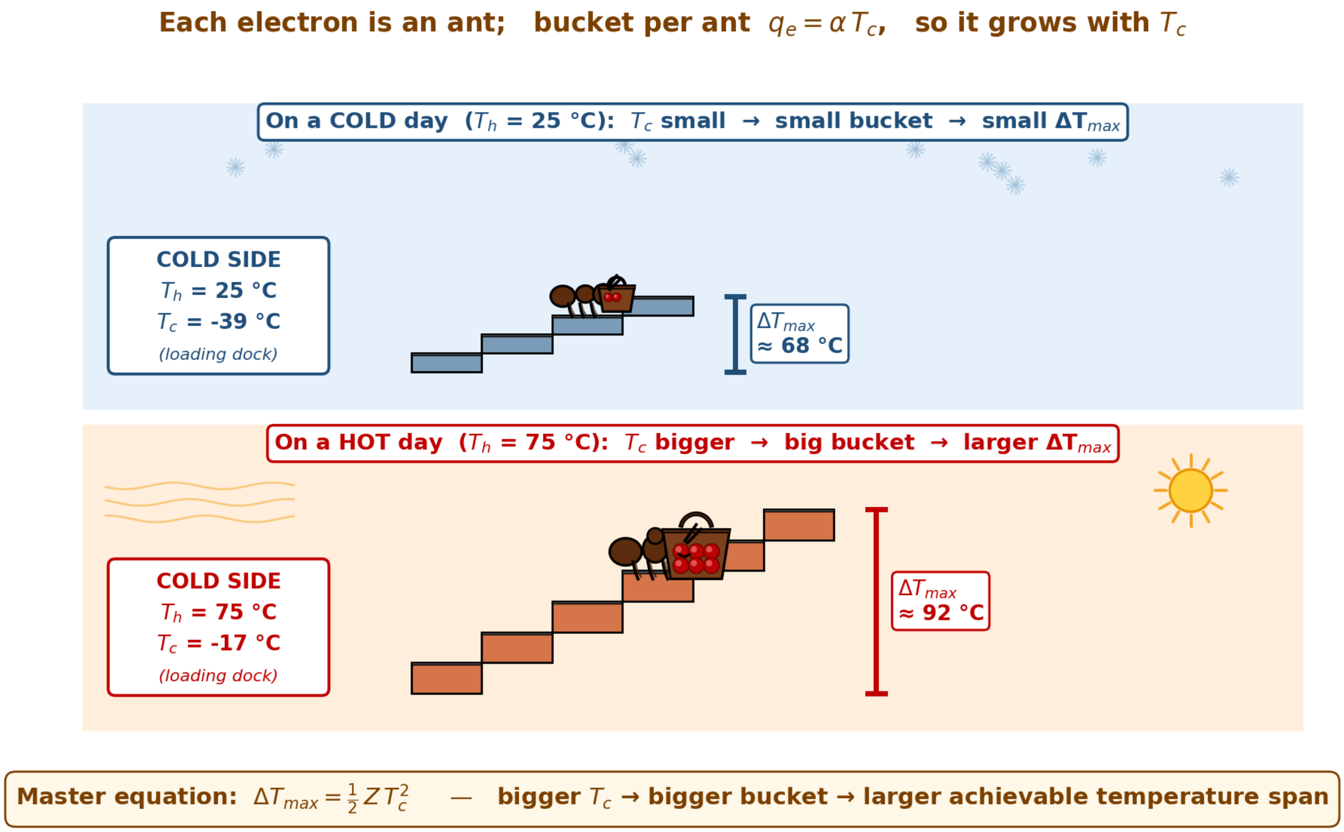

Figure 2. Ants carrying heat buckets. On a cold day each ant has a small bucket and barely moves any heat (ΔTmax ≈ 68 °C). On a hot day each ant is loaded up with a much larger bucket and the module reaches ΔTmax ≈ 92 °C — same hardware, bigger cargo per trip.

7. Expressing ΔTmax in Terms of Th

Equation (8) is in terms of Tc, but for engineering use we need it in terms of Th (which the heat sink fixes). Substituting Tc = Th − ΔTmax:

$$\Delta\text{Tmax} = \tfrac{1}{2}\,\text{Z}\,(\,\text{Th} - \Delta\text{Tmax}\,)^2 \qquad \textbf{(9)}$$

Expanding and rearranging into a standard quadratic in ΔTmax:

$$\tfrac{1}{2}\,\text{Z}\,\Delta\text{Tmax}^2 - (\,\text{Z Th} + 1\,)\,\Delta\text{Tmax} + \tfrac{1}{2}\,\text{Z Th}^2 = 0 \qquad \textbf{(10)}$$

The physically meaningful root (the one that keeps Tc > 0 K) is:

$$\Delta\text{Tmax} = \text{Th} + 1/\text{Z} - \sqrt{(\,2\text{Th}/\text{Z} + 1/\text{Z}^2\,)} \qquad \textbf{(11)}$$

A simplified approximation, valid only when ZTh ≪ 1, is sometimes seen:

$$\Delta\text{Tmax} \approx \tfrac{1}{2}\,\text{Z Th}^2 \qquad \textbf{(12)}$$

Use Equation (11) for any quantitative work. For real Bi_2Te_3 the product ZTh is of order 1 (not ≪ 1) at typical operating temperatures, so Equation (12) overestimates the exact value substantially. It is shown here for completeness only and is not plotted in Figure 1.

8. Numerical Example and Graphical Comparison

Using $Z = 2.5 \times 10^{-3}\ K^{-1}$ (typical for Bi_2Te_3), Equation (11) gives the values in Table 1. The *normalized* columns express ΔTmax(Th) as a multiple of ΔTmax at Th = 27 °C and ITEC(Th) as a multiple of Imax at Th = 27 °C — multiply each ratio by the corresponding datasheet value to predict performance at any Th.

Table 1. ΔTmax and ITEC predictions for Bi_2Te_3 ($Z = 2.5 \times 10^{-3}\ K^{-1}$).

Th (°C)	Th (K)	Tc (K)	ΔTmax (°C)	ΔTmax / ΔTmax,ref	ITEC (A)	ITEC / Imax	Notes
-20	253.15	202.10	**51.05**	0.755	5.56	0.869	
0	273.15	215.24	**57.91**	0.857	5.92	0.926	
25	298.15	231.28	**66.87**	0.989	6.37	0.995	
27	300.15	232.55	**67.60**	**1.000**	6.40	**1.000**	**ref**
50	323.15	246.93	**76.22**	1.128	6.80	1.062	
75	348.15	262.21	**85.94**	1.271	7.22	1.128	
100	373.15	277.14	**96.01**	1.420	7.63	1.192	
120	393.15	288.85	**104.30**	1.543	7.95	1.242	

Reference row at Th = 27 °C anchors both normalized columns. Multiply each ratio by your module's datasheet ΔTmax (or Imax) to predict performance at the actual operating Th.

Figure 1 plots the same data over a continuous Th sweep from −20 °C to 120 °C. The blue curve is the normalized ΔTmax(Th) ratio (left axis). The brown dot-dashed curve is ITEC / Imax (right axis), also normalized to the Th = 27 °C reference. The red dots mark the datasheet anchor where both curves equal 1.000 by definition.

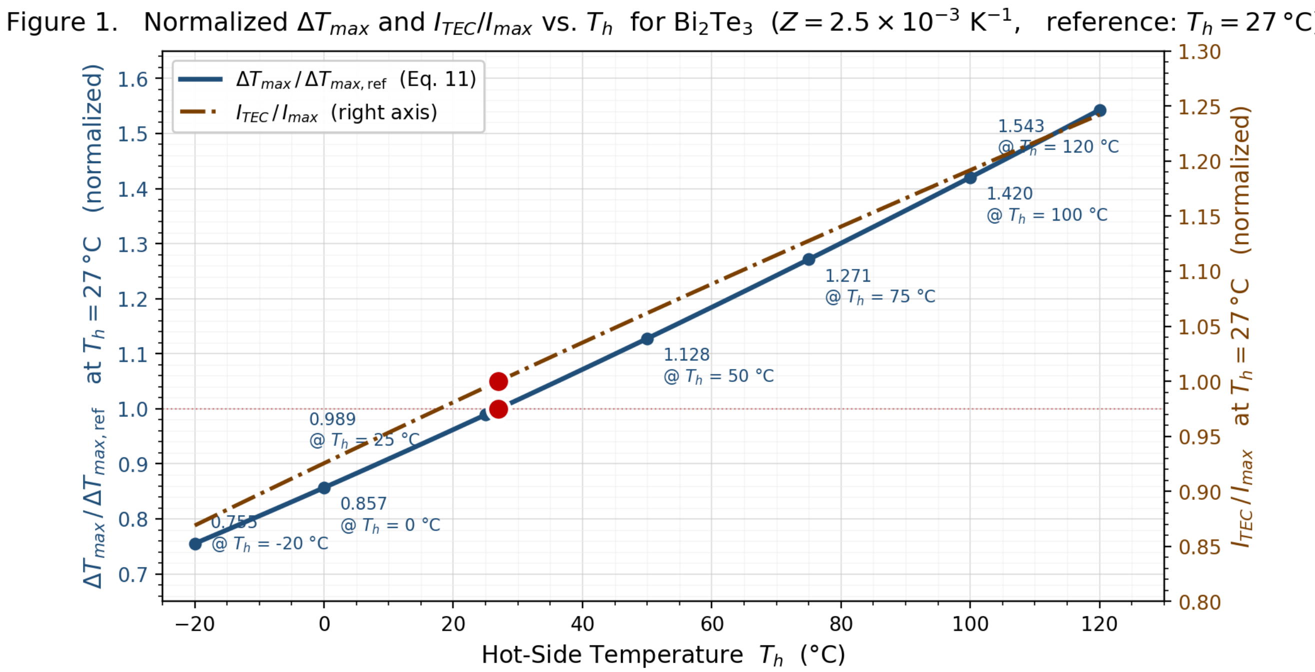

Figure 1. Normalized ΔTmax (left axis) and ITEC / Imax (right axis) vs. Th for Bi_2Te_3, $Z = 2.5 \times 10^{-3}\ K^{-1}$, reference Th = 27 °C.

9. The TEC Drive Current at ΔTmax

Equation (5) gives Iopt = α Tc / R, and Tc rises with Th. In Figure 1 the brown dot-dashed curve shows ITEC / Imax swinging from about 0.87 at Th = −20 °C up to 1.24 at Th = 120 °C — a 40 % swing. Two practical implications follow:

1. Drive-current sizing. A controller that current-limits to the datasheet Imax value at 27 °C will under-drive the module at Th = 100 °C and miss the larger ΔTmax that the physics would otherwise allow. ATI controllers (TEC18V15A, TEC50V20ACH) support adjustable current limits via the IMS/VLM pins so the user can match Iopt to the actual operating Th.

2. Power-supply sizing. The TEC voltage at Iopt is roughly Vopt ≈ α Tc + Iopt R ≈ 2 α Tc, so peak input power demand at high Th is materially larger than at room temperature. Always size VPS for the worst-case (highest-Th) operating point.

10. Practical Implications for System Designers

Specify ΔTmax at the actual operating Th, not the datasheet reference. Datasheet ΔTmax values are quoted at Th = 27 °C by convention, but most real systems run hotter. Read the normalization ratio from Figure 1 (or Table 1), multiply by your datasheet value, and use that for design.

Use Equation (11), not Equation (12). The simplified parabolic form overshoots the exact value substantially for any modern TE material at typical Th, so it is omitted from Figure 1 and should be reserved for back-of-envelope estimates only.

Match Iopt to Th. If the system runs over a wide ambient range, set the controller current limit to track Iopt(Th) using a Th sense input. This squeezes out the last 5 – 15 % of ΔTmax that a fixed current limit leaves on the table.

Keep Tc above the dew point if condensation is a concern. In humid environments, the cold side at ΔTmax can drop below the dew point and accumulate water. Run the module sub-ΔTmax, or seal/desiccate the cold-side enclosure.

11. Summary

The maximum temperature difference of a thermoelectric cooler is governed by the cooling-side master equation $\Delta T_{max} = \frac{1}{2} Z T_c^2$. Because Tc rises with Th at the ΔTmax operating point, the achievable temperature span itself rises with Th. Each electron carries Peltier heat αTc per coulomb, so the 'bucket per ant' grows with absolute temperature; that is the whole story behind the Th-dependence.

Equation (11) gives the exact closed-form ΔTmax(Th); use it. At the optimum operating point the drive current $I_{opt} = \alpha T_c / R$ also rises with Th — size your controller and supply accordingly.

References

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[3] Analog Technologies, Inc., TEC Modules Product Line, www.analogtechnologies.com/tec-module.html.

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